General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Wednesday 21 January 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 4x^3 - 7x - 3$.

(i) Find $\mathrm{f}(-1)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $2x + 1$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(iii) Simplify the algebraic fraction $\dfrac{4x^3 - 7x - 3}{2x^2 + 3x + 1}$. *(3 marks)*

(b) The polynomial $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 4x^3 - 7x + d$. When $\mathrm{g}(x)$ is divided by $2x + 1$, the remainder is 2. Find the value of d. *(2 marks)*

2 (a) Express $\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. Give your value of α in radians to two decimal places. *(3 marks)*

(b) Hence:

(i) write down the minimum value of $\sin x - 3\cos x$; *(1 mark)*

(ii) find the value of x in the interval $0 < x < 2\pi$ at which this minimum value occurs, giving your value of x in radians to two decimal places. *(2 marks)*

3 (a) (i) Express $\dfrac{2x + 7}{x + 2}$ in the form $A + \dfrac{B}{x + 2}$, where A and B are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{2x + 7}{x + 2}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Express $\dfrac{28 + 4x^2}{(1 + 3x)(5 - x)^2}$ in the form $\dfrac{P}{1 + 3x} + \dfrac{Q}{5 - x} + \dfrac{R}{(5 - x)^2}$, where P, Q and R are constants. *(5 marks)*

(ii) Hence find $\displaystyle\int \frac{28 + 4x^2}{(1 + 3x)(5 - x)^2}\,\mathrm{d}x$. *(4 marks)*

4 (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence obtain the binomial expansion of $\sqrt{4-x}$ up to and including the term in x^2. *(3 marks)*

(b) Use your answer to part (a)(ii) to find an approximate value for $\sqrt{3}$. Give your answer to three decimal places. *(2 marks)*

5 (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(b) Solve the equation

$$5\sin 2x + 3\cos x = 0$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$ to the nearest 0.1°, where appropriate. *(4 marks)*

(c) Given that $\sin 2x + \cos 2x = 1 + \sin x$ and $\sin x \neq 0$, show that $2(\cos x - \sin x) = 1$. *(4 marks)*

6 A curve is defined by the equation $x^2y + y^3 = 2x + 1$.

(a) Find the gradient of the curve at the point $(2, 1)$. *(6 marks)*

(b) Show that the x-coordinate of any stationary point on this curve satisfies the equation

$$\frac{1}{x^3} = x + 1$$

(4 marks)

Turn over for the next question

7 (a) A differential equation is given by $\frac{\mathrm{d}x}{\mathrm{d}t} = -kte^{\frac{1}{2}x}$, where k is a positive constant.

(i) Solve the differential equation. *(3 marks)*

(ii) Hence, given that $x = 6$ when $t = 0$, show that $x = -2\ln\left(\frac{kt^2}{4} + e^{-3}\right)$. *(3 marks)*

(b) The population of a colony of insects is decreasing according to the model $\frac{\mathrm{d}x}{\mathrm{d}t} = -kte^{\frac{1}{2}x}$, where x thousands is the number of insects in the colony after time t minutes. Initially, there were 6000 insects in the colony.

Given that $k = 0.004$, find:

(i) the population of the colony after 10 minutes, giving your answer to the nearest hundred; *(2 marks)*

(ii) the time after which there will be no insects left in the colony, giving your answer to the nearest 0.1 of a minute. *(2 marks)*

8 The points A and B have coordinates $(2,\ 1,\ -1)$ and $(3,\ 1,\ -2)$ respectively. The angle OBA is θ, where O is the origin.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that $\cos\theta = \frac{5}{2\sqrt{7}}$. *(4 marks)*

(b) The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. The line l is parallel to $\overrightarrow{AB}$ and passes through the point C. Find a vector equation of l. *(2 marks)*

(c) The point D lies on l such that angle $ODC = 90°$. Find the coordinates of D. *(4 marks)*

END OF QUESTIONS